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A Generalized Ohm's Law of Plasma

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CONTENTS

I. Introduction	1
II. Generalization of the Chapman — Cowling Discussion of Current Density	3
III. An Alternative Derivation from the Thermodynamics of the Irreversible Process	13
IV. Conclusions	21
References	23

ABSTRACT

A generalized Ohm's law is derived for ionized gas in the presence of electrical and magnetic fields with arbitrary orientations. Two different methods are used in the analysis but the results are in complete agreement. It is found that in general when the magnetic field and the electrical field are neither parallel nor perpendicular, an additional current normal to both the direct current and the Hall current is theoretically observed.

I. INTRODUCTION

It has been realized for a long time that only under certain special conditions may the classical Ohm's law – which states that the current is proportional and parallel to the electrical field, or in mathematical terms simply $\mathbf{J} = \sigma \mathbf{E}$ – be used in the study of ionized gas. For the generalization of this classical Ohm's law, contributions have been given by many authors, for example, Chapman, Cowling, Spitzer, and Finkelnburg and Maecker (Ref. 1-4). Generalization may be generally considered in two different directions; one is to extend the definition of \mathbf{E} as the effective electrical field which may include the field generated by the diffusion phenomena due to pressure gradient and concentration gradient and the other is to generalize the classical Ohm's law in such a way that the Hall effect may be taken into account. In Ref. 1, a fairly general expression of electrical current is derived on the basis of kinetic theory. However, only two special cases are discussed in detail: in one case the magnetic field \mathbf{B} is assumed to be parallel to \mathbf{E} , and in the other \mathbf{B} is perpendicular to \mathbf{E} . In Ref. 2, Cowling has derived a generalized Ohm's law in an implicit form

$$\mathbf{J} + \omega \tau^{-1} \mathbf{J} \times \mathbf{B} = \sigma \mathbf{E}' \quad (1)$$

based on a much simpler theory. However, again only two special cases are discussed: $\mathbf{B} \parallel \mathbf{E}$ and $\mathbf{B} \perp \mathbf{E}$. Spitzer's expression of Ohm's law (Ref. 3) takes into account the unsteady effect, and his

result agrees with Cowling's in the steady-state case. The result derived by Finkelburg and Maecker (Ref. 4) also expresses the electrical current in an implicit form, namely

$$\mathbf{J} = \sigma \left[\mathbf{E} + \frac{1}{c} (\mathbf{v} \times \mathbf{H}) - \frac{1}{enc} (\mathbf{J} \times \mathbf{H}) + \frac{m_e m_i}{e(m_e + m_i)} \left(\frac{1}{\rho_e} \text{grad } p_e - \frac{1}{\rho_i} \text{grad } p_i \right) \right] \quad (2)$$

The purpose of the present paper is to derive a more general and explicit expression of the electrical current \mathbf{J} . Two different methods will be used. The first method will be shown in Section II, which follows the technique used in Ref. 1 but will remove the assumption that \mathbf{B} is perpendicular to \mathbf{E} . The result gives some novel features besides the classical Hall effect. The second method, which is based on the irreversible process of thermodynamics, will be demonstrated in Section III. The final result is obtained in a form different from that given in Ref. 4 but agrees with what is shown in Section II. This explicit expression of current density \mathbf{J} may be considered as a generalized Ohm's law which takes the form

$$\mathbf{J} = \sigma \frac{[\mathbf{E}' + \omega \tau \mathbf{B}^{-1} (\mathbf{B} \times \mathbf{E}') + \omega^2 \tau^2 \mathbf{B}^{-2} \mathbf{B} (\mathbf{B} \cdot \mathbf{E}')] }{1 + \omega^2 \tau^2}$$

II. GENERALIZATION OF THE CHAPMAN — COWLING DISCUSSION OF CURRENT DENSITY

Considering an ionized gas of two charged species, Chapman and Cowling (Ref. 1) discussed the solution of the Boltzmann equation to the second approximation. If f_1 and f_2 are the distribution functions of the two species, then f_1 and f_2 satisfy

$$\frac{\partial f_1}{\partial t} + \mathbf{v}_1 \cdot \frac{\partial f_1}{\partial \mathbf{r}} + \frac{e_1}{m_1} (\mathbf{E} + \mathbf{v}_1 \times \mathbf{B}) \cdot \frac{\partial f_1}{\partial \mathbf{v}_1} = -J_1(ff_1) - J_{12}(f_1 f_2) \quad (3)$$

$$\frac{\partial f_2}{\partial t} + \mathbf{v}_2 \cdot \frac{\partial f_2}{\partial \mathbf{r}} + \frac{e_2}{m_2} (\mathbf{E} + \mathbf{v}_2 \times \mathbf{B}) \cdot \frac{\partial f_2}{\partial \mathbf{v}_2} = -J_2(ff_2) - J_{21}(f_2 f_1) \quad (4)$$

where $J_1(ff_1)$ is the collision integral which describes the change of f_1 due to the interaction between particles of species 1 and $J_{12}(f_1 f_2)$ is due to the interaction between particles of species 1 and 2. The integrals $J_2(ff_2)$ and $J_{21}(f_2 f_1)$ are similarly defined for f_2 . Postulating that the distribution function f_1 may be written in the form $f_1 = f_1^{(0)} + f_1^{(1)}$, where $f_1^{(1)}$ is a second approximation which is much smaller than the first approximation $f_1^{(0)}$, two successive approximations may be obtained from Eq. (3).

$$\frac{e_1}{m_1} \mathbf{v}_1 \times \mathbf{B} \cdot \frac{\partial f_1^{(0)}}{\partial \mathbf{v}_1} = -J_1(f_1^{(0)} f_1^{(0)}) - J_{12}(f_1^{(0)} f_2^{(0)}) \quad (5)$$

$$\begin{aligned} \frac{D_0 f_1^{(0)}}{Dt} + \mathbf{v}_1 \cdot \frac{\partial f_1^{(0)}}{\partial \mathbf{r}} + \left[\frac{e_1}{m_1} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \frac{D_0 \mathbf{v}}{Dt} \right] \cdot \frac{\partial f_1^{(0)}}{\partial \mathbf{v}_1} \\ + \frac{e_1}{m_1} (\mathbf{v}_1 \times \mathbf{B}) \cdot \frac{\partial f_1^{(1)}}{\partial \mathbf{v}_1} - \frac{\partial f_1^{(0)}}{\partial \mathbf{v}_1} \cdot \mathbf{v}_1 \cdot \frac{\partial \mathbf{v}}{\partial \mathbf{r}} \\ = -J_1(f_1^{(0)} f_1^{(1)}) - J_1(f_1^{(1)} f_1^{(0)}) - J_{12}(f_1^{(1)} f_2^{(0)}) - J_{12}(f_1^{(0)} f_2^{(1)}) \end{aligned} \quad (6)$$

The notations used hereafter are mostly adopted from Ref. 1, except the velocities and the fields. \mathbf{V}_1 denotes the peculiar velocity and \mathbf{v} the velocity of the mixture (in Chapman-Cowling notation $\mathbf{V}_1 = \mathbf{C}_1$ and $\mathbf{v} = \mathbf{c}_0$). The differential operator D_0/Dt is defined in Ref. 1, p. 113. The solution of $f_1^{(0)}$ may be found as

$$f_1^{(0)} = n_1 \left(\frac{m_1}{2\pi KT} \right)^{\frac{3}{2}} e^{-m_1 \mathbf{V}_1^2 / 2KT} \quad (7)$$

where n_1 and T are the number density and temperature of the first gas. Now, rewriting $f_1^{(1)} = f_1^{(0)} \Phi_1^{(1)}$ and using the relations

$$\frac{D_0 n_1}{Dt} + n_1 \frac{\partial \mathbf{v}}{\partial \mathbf{r}} = 0 \quad (8)$$

$$\rho \frac{D_0 \mathbf{v}}{Dt} = (n_1 e_1 + n_2 e_2) (\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \mathbf{J}^{(1)} \times \mathbf{B} - \frac{\partial p}{\partial \mathbf{r}} \quad (9)$$

$$\frac{3}{2} Kn \frac{D_0 T}{Dt} = -KnT \frac{\partial \mathbf{v}}{\partial \mathbf{r}} \quad n = n_1 + n_2 \quad (10)$$

Eq. (4) may be put in the form

$$\begin{aligned} f_1^{(0)} & \left[\frac{m_1}{KT} \mathbf{V}_1^0 \mathbf{V}_1 : \frac{\partial \mathbf{v}}{\partial \mathbf{r}} + \mathbf{V}_1 \left(\frac{m_1 V_1^2}{2KT} - \frac{5}{2} \right) \cdot \frac{1}{T} \frac{\partial T}{\partial \mathbf{r}} + \frac{n}{n_1} \mathbf{d}_{12} \cdot \mathbf{V}_1 \right] \\ & = -f_1^{(0)} \left[\frac{m_1}{\rho KT} \mathbf{V}_1 \cdot (\mathbf{J}^{(1)} \times \mathbf{B}) + \frac{e_1}{m_1} (\mathbf{V}_1 \times \mathbf{B}) \cdot \frac{\partial \Phi_1^{(1)}}{\partial \mathbf{V}_1} \right] \\ & \quad - n_1^2 I_1(\Phi_1^{(1)}) - n_1 n_2 I_{12}(\Phi_1^{(1)} + \Phi_2^{(1)}) \end{aligned} \quad (11)$$

where d_{12} is a vector quantity which is defined as

$$d_{12} = \frac{\partial n_{10}}{\partial r} + \frac{n_1 n_2 (m_2 - m_1)}{n \rho p} \frac{\partial p}{\partial r} - \frac{\rho_1 \rho_2}{\rho p} \left(\frac{e_1}{m_1} - \frac{e_2}{m_2} \right) \mathbf{E} - \frac{n_1 n_2}{\rho p} (m_2 e_1 - m_1 e_2) \mathbf{v} \times \mathbf{B}$$

$$J^{(1)} = n_1 e_1 \langle \mathbf{V}_1^{(1)} \rangle + n_2 e_2 \langle \mathbf{V}_2^{(1)} \rangle$$

$$n_1 \langle \mathbf{V}_1^{(1)} \rangle = \int f_1^{(1)} \mathbf{V}_1 d\mathbf{V}_1$$

$$n_{10} = \frac{n_1}{n_1 + n_2} = \frac{n_1}{n}$$

and the operators I_1 and I_{12} are defined in Ref. 1, p. 85. An equation similar to Eq. (11) may be derived for the second gas simply by changing the sign of d_{12} . Thus

$$\begin{aligned} f_2^{(0)} & \left[\frac{m_2}{KT} \mathbf{V}_2^0 \mathbf{V}_2 : \frac{\partial \mathbf{v}}{\partial r} + \mathbf{V}_2 \cdot \left(\frac{m_2 V_2^2}{\rho KT} - \frac{5}{2} \right) \frac{1}{T} \frac{\partial T}{\partial r} - \frac{n_1}{n_2} d_{12} \cdot \mathbf{V}_2 \right] \\ & = -f_2^{(0)} \left[\frac{m_2}{\rho KT} \mathbf{V}_2 \cdot (J^{(1)} \times \mathbf{B}) + \frac{e_1}{m_1} (\mathbf{V}_2 \times \mathbf{B}) \cdot \frac{\partial \Phi_2^{(1)}}{\partial \mathbf{V}_2} \right] \\ & \quad - n_2^2 I_2 (\Phi_2^{(1)}) - n_1 n_2 I_{21} (\Phi_2^{(1)}) + \Phi_1^{(1)} \end{aligned} \quad (12)$$

According to Chapman and Cowling,

$$\Phi_1^{(1)} = -\mathbf{A}_1 \cdot \frac{1}{r} \frac{\partial T}{\partial r} - \bar{B}_1 : \frac{\partial \mathbf{v}}{\partial r} - n_1 \mathbf{D}_1 \cdot d_{12} \quad (13)$$

$$\Phi_2^{(1)} = -A_2 \cdot \frac{1}{r} \frac{\partial T}{\partial r} - \bar{B}_2 \cdot \frac{\partial \mathbf{v}}{\partial r} - n_1 \mathbf{D}_1 \cdot \mathbf{d}_{12} \quad (14)$$

The quantities A , \bar{B} and \mathbf{D} are to be determined. Since the purpose of the present discussion is to understand the electrical current density due to diffusion¹, only the vector \mathbf{D} shall be studied. In general \mathbf{D}_1 is a function of \mathbf{V}_1 and \mathbf{B} . It is argued in Ref. 1 that since only the vector product of $\mathbf{V} \times \mathbf{B}$ appears in the mechanical equations, \mathbf{D}_1 can only depend upon \mathbf{V}_1 , $\mathbf{V}_1 \times \mathbf{B}$, $(\mathbf{V}_1 \times \mathbf{B}) \times \mathbf{B}$, etc. Therefore, it is justifiable to write

$$\mathbf{D}_1 = D_1^I \mathbf{V}_1 + D_1^{II} (\mathbf{V}_1 \times \mathbf{B}) + D_1^{III} (\mathbf{V}_1 \times \mathbf{B}) \times \mathbf{B}$$

where the unknown coefficients D_1^I , D_1^{II} , and D_1^{III} are functions of the magnitude of \mathbf{V}_1 and \mathbf{B} . To discuss the diffusion phenomena only, it is possible to write

$$\Phi_1^{(1)} = -n \mathbf{d}_{12} \cdot [D_1^I \mathbf{V}_1 + D_1^{II} (\mathbf{V}_1 \times \mathbf{B}) + D_1^{III} (\mathbf{V}_1 \times \mathbf{B}) \times \mathbf{B}]$$

But

$$(\mathbf{V}_1 \times \mathbf{B}) \times \mathbf{B} = \mathbf{B} (\mathbf{B} \cdot \mathbf{V}_1) - B^2 \mathbf{V}_1$$

There is, therefore, no loss of generality, if the expression is rewritten

$$\Phi_1^{(1)} = -n \mathbf{d}_{12} \cdot [D_1^I \mathbf{V}_1 + D_1^{II} (\mathbf{V}_1 \times \mathbf{B}) + D_1^{III} \mathbf{B} (\mathbf{B} \cdot \mathbf{V}_1)] \quad (15)$$

Similarly

$$\Phi_2^{(1)} = -n \mathbf{d}_{12} \cdot [D_2^I \mathbf{V}_2 + D_2^{II} (\mathbf{V}_2 \times \mathbf{B}) + D_2^{III} \mathbf{B} (\mathbf{B} \cdot \mathbf{V}_2)] \quad (16)$$

With (15) and (16), the current density $\mathbf{J}^{(1)}$ may be expressed in terms of $\Phi_1^{(1)}$ and $\Phi_2^{(1)}$ as follows

$$\begin{aligned} \mathbf{J}^{(1)} &= e_1 \int f_1^{(0)} \Phi_1^{(1)} \mathbf{V}_1 d\mathbf{V}_1 + e_2 \int f_2^{(0)} \Phi_2^{(1)} \mathbf{V}_2 d\mathbf{V}_1 \\ &= -n [L^I \mathbf{d}_{12} + L^{II} \mathbf{B} \times \mathbf{d}_{12} + L^{III} \mathbf{B} (\mathbf{B} \cdot \mathbf{d}_{12})] \end{aligned} \quad (17)$$

¹ For the time being, the thermal diffusion will be ignored.

where

$$L_1^I = \frac{e_1}{3} \int D_1^I f_1^{(0)} V_1^2 d\mathbf{V}_1 + \frac{e_2}{3} \int D_2^I f_2^{(0)} V_2^2 d\mathbf{V}_2$$

$$L_1^{II} = - \left[\frac{e_1}{3} \int D_1^{II} f_1^{(0)} V_1^2 d\mathbf{V}_1 + \frac{e_2}{3} \int D_2^{II} f_2^{(0)} V_2^2 d\mathbf{V}_2 \right]$$

$$L_1^{III} = \frac{e_1}{3} \int D_1^{III} f_1^{(0)} V_1^2 d\mathbf{V}_1 + \frac{e_2}{3} \int D_2^{III} f_2^{(0)} V_2^2 d\mathbf{V}_2$$

Substituting Eq. (15), (16), and (17) into the equation

$$\begin{aligned} \frac{n}{n_1} f_1^{(0)} \mathbf{d}_{12} \cdot \mathbf{V}_1 = & -f_1^{(0)} \left\{ \frac{m_1}{\rho K T} \mathbf{V}_1 \cdot (\mathbf{J}^{(1)} \times \mathbf{B}) + \frac{e_1}{m_1} (\mathbf{V}_1 \times \mathbf{B}) \cdot \frac{\partial \Phi_1^{(1)}}{\partial \mathbf{V}_1} \right\} \\ & - \left\{ n_1^2 I_1 (\Phi_1^{(1)}) + n_1 n_2 I_{12} (\Phi_1^{(1)} + \Phi_2^{(1)}) \right\} \end{aligned} \quad (18)$$

which is the reduced form of Eq. (11), and equating the terms having coefficients \mathbf{d}_{12} , $\mathbf{B} \times \mathbf{d}_{12}$ and $\mathbf{B}(\mathbf{B} \cdot \mathbf{d}_{12})$, three equations are obtained as follows:

$$\begin{aligned} \frac{1}{n_1} f_1^{(0)} \mathbf{V}_1 = & -f_1^{(0)} \left\{ - \frac{m_1}{\rho K T} L_1^{II} B^2 \mathbf{V}_1 + \frac{e_1}{m_1} D_1^{II} B^2 \mathbf{V}_1 \right\} + n_1^2 I_1 (D_1^I \mathbf{V}_1) \\ & + n_1 n_2 I_{12} (D_1^I \mathbf{V}_1 + D_2^I \mathbf{V}_2) \end{aligned} \quad (19)$$

$$0 = -f_1^{(0)} \left\{ - \frac{m_1}{\rho K T} L_1^I \mathbf{V}_1 + \frac{e_1}{m_1} D_1^I \mathbf{V}_1 \right\} + n_1^2 I_1 (D_1^{II} \mathbf{V}_1) + n_1 n_2 I_{12} (D_1^{II} \mathbf{V}_1 + D_2^{II} \mathbf{V}_2) \quad (20)$$

$$0 = f_1^{(0)} \left\{ \frac{e_1}{m_1} D_1^{II} \mathbf{V}_1 \right\} + n_1^2 I_1 (D_1^{III} \mathbf{V}_1) + n_1 n_2 I_{12} (D_1^{III} \mathbf{V}_1 + D_2^{III} \mathbf{V}_2) \quad (21)$$

velocity space. Thus

$$\begin{aligned} \frac{1}{n_1} \int f_1^{(0)} \mathbf{v}_1 \mathbf{a}_1^{(0)} d\mathbf{v}_1 &= \frac{1}{\rho K T} L_1^{\text{II}} B^2 \int m_1 f_1^{(0)} \mathbf{a}_1^{(0)} \mathbf{v}_1 d\mathbf{v}_1 - \frac{e_1}{m_1} B^2 \int f_1^{(0)} \epsilon'_0 \mathbf{a}_1^{(0)} \cdot \mathbf{a}_1^{(0)} d\mathbf{v}_1 \\ &+ \int [n_1^2 I_1(D_1^I \mathbf{v}_1) + n_1 n_2 I_{12}(D_1^I \mathbf{v}_1 + D_2^I \mathbf{v}_2)] \mathbf{a}_2^{(0)} d\mathbf{v}_1 \end{aligned} \quad (26)$$

$$\begin{aligned} - \frac{1}{n_2} \int f_2^{(0)} \mathbf{v}_2 \mathbf{a}_2^{(0)} d\mathbf{v}_2 &= \frac{1}{\rho K T} L_2^{\text{II}} B^2 \int m_2 f_2^{(0)} \mathbf{a}_2 \mathbf{v}_2 d\mathbf{v}_2 - \frac{e_2}{m_2} \int f_2^{(0)} \epsilon'_0 \mathbf{a}_2^{(0)} \cdot \mathbf{a}_2^{(0)} d\mathbf{v}_2 \\ &+ \int [n_2^2 I_2(D_2^I \mathbf{v}_2) + n_1 n_2 I_{21}(D_2^I \mathbf{v}_2 + D_1^I \mathbf{v}_1)] \mathbf{a}_1^{(0)} d\mathbf{v}_2 \end{aligned} \quad (27)$$

Adding Eq. (27) to (26) yields

$$(2KT)^{1/2} = - \frac{n_1 n_2 \epsilon'_0 B^2}{\rho^2} (e_1 m_2 \rho_2 + e_2 m_1 \rho_1) + KT \frac{n_1 n_2 \epsilon_0}{n(D_{12})_1} \quad (28)$$

because

$$\frac{1}{n_1} \int f_1^{(0)} \mathbf{v}_1 \mathbf{a}_1^{(0)} d\mathbf{v}_1 - \frac{1}{n_2} \int f_2^{(0)} \mathbf{v}_2 \mathbf{a}_2^{(0)} d\mathbf{v}_2 = \frac{3}{2} (2KT)^{1/2}$$

$$m_1 \int f_1^{(0)} \mathbf{v}_1 \mathbf{a}_1^{(0)} d\mathbf{v}_1 + m_2 \int f_2^{(0)} \mathbf{v}_2 \mathbf{a}_2^{(0)} d\mathbf{v}_2 = 0$$

$$\frac{e_1}{m_1} B^2 \int f_1^{(0)} \epsilon'_0 \mathbf{a}_1^{(0)} \cdot \mathbf{a}_1^{(0)} d\mathbf{v}_1 + \frac{e_2}{m_2} B^2 \int f_2^{(0)} \epsilon'_0 \mathbf{a}_2^{(0)} \cdot \mathbf{a}_2^{(0)} d\mathbf{v}_2$$

$$= \frac{3}{2} B^2 \frac{\rho_1^2 \rho_2^2}{\rho^2} \left[\frac{e_1 n_1}{n_1^2 m_1^2} + \frac{e_2 n_2}{n_2^2 m_2^2} \right] \epsilon'_0$$

Similarly for the second species:

$$-\frac{1}{n_1} f_2^{(0)} \mathbf{V}_2 = -f_2^{(0)} \left\{ -\frac{m_2}{\rho K T} L_2^{\text{II}} B^2 \mathbf{V}_1 + \frac{e_1}{m_1} D_2^{\text{II}} B^2 \mathbf{V}_1 \right\} + n_2^2 I_2 (D_2^{\text{I}} \mathbf{V}_2) \\ + n_2 n_1 I_{21} (D_2^{\text{I}} \mathbf{V}_2 + D_2^{\text{I}} \mathbf{V}_1) \quad (22)$$

$$0 = f_2^{(0)} \left\{ -\frac{m_1}{\rho K T} L_2^{\text{I}} \mathbf{V}_2 + \frac{e_2}{m_2} D_2^{\text{I}} \mathbf{V}_2 \right\} + n_2^2 I_2 (D_2^{\text{II}} \mathbf{V}_2) + n_1 n_2 I_{21} (D_1^{\text{II}} \mathbf{V}_1 + D_2^{\text{II}} \mathbf{V}_2) \quad (23)$$

$$0 = f_2^{(0)} \left\{ \frac{e_2}{m_2} D_2^{\text{II}} \mathbf{V}_2 \right\} + n_2^2 I_2 (D_2^{\text{III}} \mathbf{V}_1) + n_1 n_2 I_{21} (D_1^{\text{III}} \mathbf{V}_1 + D_2^{\text{III}} \mathbf{V}_2) \quad (24)$$

Following the same approximation used by Chapman and Cowling in discussing the special case that \mathbf{B} is perpendicular to \mathbf{E} , the first approximation is

$$\left. \begin{aligned} D_1^{\text{I}} \mathbf{V}_1 &= \epsilon_0 \mathbf{a}_1^{(0)} & D_2^{\text{I}} \mathbf{V}_2 &= \epsilon_0 \mathbf{a}_2^{(0)} \\ D_1^{\text{II}} \mathbf{V}_1 &= \epsilon'_0 \mathbf{a}_1^{(0)} & D_2^{\text{II}} \mathbf{V}_2 &= \epsilon'_0 \mathbf{a}_2^{(0)} \\ D_1^{\text{III}} \mathbf{V}_1 &= \epsilon''_0 \mathbf{a}_1^{(0)} & D_2^{\text{III}} \mathbf{V}_2 &= \epsilon''_0 \mathbf{a}_2^{(0)} \end{aligned} \right\} \quad (25)$$

where the vectors \mathbf{a}_1 and \mathbf{a}_2 have been originally introduced in Ref. 1, and in first approximation $\mathbf{a}_1^{(0)}$ and $\mathbf{a}_2^{(0)}$ may be defined as

$$\mathbf{a}_1^{(0)} = \frac{\rho_1 \rho_2}{\rho n_1} \sqrt{\frac{1}{2KT}} \mathbf{V}_1 \quad \mathbf{a}_2^{(0)} = \frac{\rho_1 \rho_2}{\rho n_2} \sqrt{\frac{1}{2KT}} \mathbf{V}_2$$

In order to determine the unknown coefficients ϵ_0 , ϵ'_0 , and ϵ''_0 , Eq. (19) is first multiplied by $\mathbf{a}_1^{(0)}$, and Eq. (22) by $\mathbf{a}_2^{(0)}$, and integrated with respect to \mathbf{V}_1 and \mathbf{V}_2 respectively throughout the

and

$$\begin{aligned} & \int [n_1^2 I_1 (D_1^I \mathbf{V}_1) + n_1 n_2 I_{12} (D_1^I \mathbf{V}_1 + D_2^I \mathbf{V}_2)] \mathbf{a}_1^{(0)} d\mathbf{V}_1 \\ & + \int [n_2^2 I_2 (D_2^I \mathbf{V}_2) + n_1 n_2 I_{21} (D_2^I \mathbf{V}_2 + D_1^I \mathbf{V}_1)] \mathbf{a}_2^{(0)} d\mathbf{V}_2 \\ & = n_1 n_2 \epsilon_0 [\mathbf{a}^{(0)} \cdot \mathbf{a}^{(0)}] = \frac{3}{2} \frac{n_1 n_2}{n} \frac{KT \epsilon_0}{(D_{12})_1} \end{aligned}$$

Following Chapman and Cowling, denote²

$$- \frac{B(e_1 m_2 \rho_2 + e_2 m_1 \rho_1)}{m_1 m_2 \rho} = \omega \text{ and } \frac{m_1 m_2 n (D_{12})_1}{\rho KT} = \tau$$

Then Eq. (28) becomes

$$(2KT)^{1/2} = -\epsilon'_0 \frac{\rho_1 \rho_2}{\rho} (B\omega) + \epsilon_0 \frac{\rho_1 \rho_2}{\rho} \frac{1}{\tau} = \frac{\rho_1 \rho_2}{\rho} \left[+\epsilon'_0 B\omega + \epsilon_0 \frac{1}{\tau} \right] \quad (29)$$

By adopting the similar technique, it is readily shown from Eq. (20) - (24) that

$$-B^{-1} \epsilon_0 \omega + \epsilon'_0 \frac{1}{\tau} = 0 \quad (30)$$

$$-B^{-1} \epsilon'_0 \omega + \epsilon''_0 \frac{1}{\tau} = 0 \quad (31)$$

Substituting $\epsilon'_0 = +B^{-1} \epsilon_0 \omega \tau$ into Eq. (29) and solving for ϵ_0 :

$$\epsilon_0 = \frac{(2KT)^{1/2} \rho}{\rho_1 \rho_2} \frac{\tau}{1 + \omega^2 \tau^2} \quad (32)$$

²The ω in the following is defined slightly different from that defined in the Reference because, in general, $[e_1 m_2 \rho_2 + e_2 m_1 \rho_1] < 0$.

Hence ϵ'_0 and ϵ''_0 may be solved from Eq. (30) and (31) immediately:

$$\epsilon'_0 = + \frac{(2KT)^{1/2} \rho}{\rho_1 \rho_2} \left(\frac{B^{-1} \omega \tau^2}{1 + \omega^2 \tau^2} \right) \quad (33)$$

$$\epsilon''_0 = + \frac{(2KT)^{1/2} \rho}{\rho_1 \rho_2} \left(\frac{B^{-2} \omega^2 \tau^3}{1 + \omega^2 \tau^2} \right) \quad (34)$$

Also,

$$\begin{aligned} \Phi_1^{(1)} &= -n \mathbf{d}_{12} \cdot [D_1^I \mathbf{V}_1 + D_1^{II} (\mathbf{V}_1 \times \mathbf{B}) + D_1^{III} \mathbf{B} (\mathbf{B} \cdot \mathbf{V})] \\ &= -n [\mathbf{d}_{12} \cdot \epsilon_0 \mathbf{a}_1^{(0)} + \epsilon'_0 B^{-1} \mathbf{a}_1^{(0)} \cdot (\mathbf{B} \times \mathbf{d}_{12}) + \epsilon''_0 \mathbf{a}_1^{(0)} \cdot B^{-2} \mathbf{B} (\mathbf{B} \cdot \mathbf{d}_{12})] \\ &= - \frac{n \rho (2KT)^{1/2}}{\rho_1 \rho_2} \frac{\tau}{1 + \omega^2 \tau^2} [\mathbf{d}_{12} \cdot \mathbf{a}_1^{(0)} + \omega \tau B^{-1} \mathbf{a}_1^{(0)} \cdot (\mathbf{B} \times \mathbf{d}_{12}) \\ &\quad + \omega^2 \tau^2 B^{-2} \mathbf{B} (\mathbf{B} \cdot \mathbf{d}_{12}) \cdot \mathbf{a}_1^{(0)}] \end{aligned} \quad (35)$$

Correspondingly

$$\begin{aligned} \Phi_2^{(1)} &= + \frac{n \rho (2KT)^{1/2}}{\rho_1 \rho_2} \left(\frac{\tau}{1 + \omega^2 \tau^2} \right) [\mathbf{d}_{12} \cdot \mathbf{a}_2^{(0)} + \omega \tau B^{-1} \mathbf{a}_2^{(0)} \cdot (\mathbf{B} \times \mathbf{d}_{12}) \\ &\quad + \omega^2 \tau^2 B^{-2} \mathbf{B} (\mathbf{B} \cdot \mathbf{d}_{12}) \cdot \mathbf{a}_2^{(0)}] \end{aligned} \quad (36)$$

Hence

$$\mathbf{J}^{(1)} = - \left(\frac{e_1}{m_1} - \frac{e_2}{m_2} \right) p \rho \tau \left[\frac{\mathbf{d}_{12} + \omega \tau B^{-1} (\mathbf{B} \times \mathbf{d}_{12}) + \omega^2 \tau^2 B^{-2} \mathbf{B} (\mathbf{B} \cdot \mathbf{d}_{12})}{1 + \omega^2 \tau^2} \right] \quad (37)$$

If an effective electrical field \mathbf{E}' is introduced such that

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B} + \frac{(m_1 - m_2)}{n(e_1 m_1 - e_2 m_1)} \nabla p - \frac{\rho p}{\rho_1 \rho_2} \left[\frac{m_1 m_2}{(e_1 m_2 - e_2 m_1)} \right] \nabla n_{10}$$

and a conductivity σ is defined as follows

$$\sigma = \frac{n_1 n_1}{\rho m_1 m_2} (e_1 m_2 - e_2 m_1)^2 \tau$$

then Eq. (27) may be rewritten as

$$\mathbf{J}^{(1)} = \sigma \left[\frac{\mathbf{E}' + \omega \tau \mathbf{B}^{-1} (\mathbf{B} \times \mathbf{E}') + \omega^2 \tau^2 \mathbf{B}^{-2} \mathbf{B} (\mathbf{B} \cdot \mathbf{E}')}{1 + \omega^2 \tau^2} \right] \quad (38)$$

Equation (38) is a generalized result, whereas the discussion given by Chapman and Cowling assumed that \mathbf{B} is perpendicular to \mathbf{E}' .

III. AN ALTERNATIVE DERIVATION FROM THE THERMODYNAMICS OF THE IRREVERSIBLE PROCESS

The previous result is based on classical kinetic theory and the method of solution was originally devised by Chapman and Enskog. An attempt is made in the present Section to find an alternative derivation of what has been previously obtained. The purpose of doing this is to see whether these two methods are fundamentally consistent. If they are consistent, the two different approaches should yield the same result or at least the results ought to be compatible from the physical or mathematical point of view.

It is a straightforward matter to obtain the energy equation of a gas mixture from the Maxwell-Boltzmann equation of transfer (Ref. 5); thus:

$$\rho \left[\frac{Dh}{Dt} - \frac{1}{\rho} \frac{Dp}{Dt} \right] = -\operatorname{div} \mathbf{Q} + \bar{\tau} \operatorname{grad} \mathbf{v} + \sum_{i=1}^s \mathbf{f}_i \cdot \frac{\mathbf{e}_i}{m_i} (\mathbf{E} + \langle \mathbf{v}_i \rangle \times \mathbf{B}) \quad (39)$$

where

$$\mathbf{Q} = \sum_{i=1}^s \rho_i \langle \epsilon_i \mathbf{V}_i \rangle = \text{heat flux}$$

$$\epsilon_i = \text{specific internal energy}$$

$$\mathbf{V}_i = \text{peculiar velocity of particle of species } i$$

$$h = \text{the specific enthalpy of the mixture}$$

$$\bar{\tau} = \sum_{i=1}^s \rho_i \langle \mathbf{V}_i^0 \mathbf{V}_i \rangle = \text{viscous stress tensor}^3$$

$$\langle \mathbf{v}_i \rangle = \text{averaged velocity of the particles of species } i$$

$$\mathbf{f}_i = \rho_i \langle \mathbf{V}_i \rangle = \text{diffusion flux of species } i$$

Since the main purpose of the present discussion is to compare the result obtained by irreversible thermodynamics to that obtained previously, the discussion will be restricted to the similar

³The notation $\mathbf{V}_i^0 \mathbf{V}_i$ is defined in Ref. 1, p. 18.

physical conditions that the gas contains only two species and that no chemical reaction is taking place. Then, on further application of the thermodynamic law,

$$T ds = dh - \frac{1}{\rho} d\rho - \sum_{i=1}^2 \mu_i dc_i \quad (40)$$

where

$$c_i = \frac{\rho_i}{\rho} = \text{relative concentration of species } i$$

$$\mu_i = \text{chemical potential of species } i$$

$$s = \text{specific entropy of the gas mixture}$$

the energy equation (39) may be written as

$$\rho T \frac{Ds}{Dt} = -\text{div } \mathbf{Q} - \rho \sum_{i=1}^2 \mu_i \frac{Dc_i}{Dt} + \bar{\tau} \text{grad } \mathbf{v} + \sum_{i=1}^2 \mathbf{f}_i \cdot \frac{e_i}{m_i} (\mathbf{E} + \langle \mathbf{v}_i \rangle \times \mathbf{B}) \quad (41)$$

However,

$$\rho \frac{Dc_i}{Dt} = -\text{div } \mathbf{f}_i$$

thus

$$\begin{aligned} \rho \frac{Ds}{Dt} = & -\text{div } \frac{\mathbf{Q} - \sum_{i=1}^2 \mu_i \mathbf{f}_i}{T} - \mathbf{Q} \cdot \frac{\text{grad } T}{T^2} - \frac{1}{T} \sum_{i=1}^2 \mathbf{f}_i \cdot \left[T \text{grad } \frac{\mu_i}{T} \right. \\ & \left. - \frac{e_i}{m_i} (\mathbf{E} + \langle \mathbf{v}_i \rangle \times \mathbf{B}) \right] + \frac{1}{T} \bar{\tau} \text{grad } \mathbf{v} \end{aligned} \quad (42)$$

Now, according to the theory of irreversible thermodynamics (Ref. 6), the phenomenological equations can be established in the following

$$\begin{aligned} \mathbf{f}_1 = I_{12} & \left[T \operatorname{grad} \left(\frac{\mu_1}{T} \right) - \frac{e_1}{m_1} (\mathbf{E} + \langle \mathbf{v}_i \rangle \times \mathbf{B}) \right] \\ & + I_{12} \left[T \operatorname{grad} \left(\frac{\mu_2}{T} \right) - \frac{e_2}{m_2} (\mathbf{E} + \langle \mathbf{v}_2 \rangle \times \mathbf{B}) \right] + I_{13} \frac{\operatorname{grad} T}{T} \end{aligned} \quad (43)$$

$$\begin{aligned} \mathbf{f}_2 = I_{21} & \left[T \operatorname{grad} \left(\frac{\mu_1}{T} \right) - \frac{e_1}{m_1} (\mathbf{E} + \langle \mathbf{v}_i \rangle \times \mathbf{B}) \right] \\ & + I_{22} \left[T \operatorname{grad} \left(\frac{\mu_1}{T} \right) - \frac{e_2}{m_2} (\mathbf{E} + \langle \mathbf{v}_2 \rangle \times \mathbf{B}) \right] + I_{23} \frac{\operatorname{grad} T}{T} \end{aligned} \quad (44)$$

$$\begin{aligned} \mathbf{Q} = I_{31} & \left[T \operatorname{grad} \left(\frac{\mu_1}{T} \right) - \frac{e_1}{m_1} (\mathbf{E} + \langle \mathbf{v}_i \rangle \times \mathbf{B}) \right] \\ & + I_{32} \left[T \operatorname{grad} \left(\frac{\mu_2}{T} \right) - \frac{e_2}{m_2} (\mathbf{E} + \langle \mathbf{v}_2 \rangle \times \mathbf{B}) \right] + I_{33} \frac{\operatorname{grad} T}{T} \end{aligned} \quad (45)$$

Since according to the definition of \mathbf{f}_i

$$\mathbf{f}_1 + \mathbf{f}_2 = 0$$

then

$$I_{11} = -I_{21} \quad (46)$$

$$I_{22} = -I_{12} \quad (47)$$

$$I_{13} = -I_{23} \quad (48)$$

In general, the Onsager's reciprocal relation must be imposed in such a way that

$$I_{12}(\mathbf{B}) = I_{21}(-\mathbf{B})$$

$$I_{13}(\mathbf{B}) = I_{31}(-\mathbf{B})$$

$$I_{23}(\mathbf{B}) = I_{32}(-\mathbf{B})$$

However, since I_{11} and I_{22} should be even functions of \mathbf{B} , from Eq. (46) and (47), it is concluded that I_{12} and I_{21} should also be even functions of \mathbf{B} . Therefore,

$$I_{12}(\mathbf{B}) = I_{21}(-\mathbf{B}) = I_{21}(\mathbf{B})$$

In the following discussion, since the thermal diffusion will not be considered, more detailed discussion of the phenomenological coefficients will be unnecessary. Thus, the diffusion flux \mathbf{f}_1 may be written as

$$\mathbf{f}_1 = I_{11} \nabla \mu + \left(\frac{e_2}{m_2} - \frac{e_1}{m_1} \right) \mathbf{E} + \left(\frac{e_2}{m_2} \langle \mathbf{v}_2 \rangle - \frac{e_1}{m_1} \langle \mathbf{v}_1 \rangle \right) \times \mathbf{B} \quad (49)$$

where $\mu = \mu_1 - \mu_2$. If the diffusion coefficients are defined in such a way that⁴

$$\begin{aligned} \mathbf{f}_1 = -\rho D_{12} \left[\nabla c + \frac{k_T}{T} \nabla T + \frac{k_p}{p} \nabla p + \frac{k_f}{p} \left(\frac{e_2}{m_2} - \frac{e_1}{m_1} \right) \mathbf{E} \right. \\ \left. + \frac{k_f}{p} \left(\frac{e_2}{m_2} \langle \mathbf{v}_2 \rangle - \frac{e_1}{m_1} \langle \mathbf{v}_1 \rangle \right) \times \mathbf{B} \right] \end{aligned}$$

the following relations may be deduced:

$$I_{11} = - \frac{\rho D_{12}}{\left(\frac{\partial \mu}{\partial c} \right)_{p,T}}, \quad k_p = -p \frac{\left(\frac{\partial \mu}{\partial p} \right)_{c,T}}{\left(\frac{\partial \mu}{\partial c} \right)_{p,T}}, \quad k_f = \frac{p}{\left(\frac{\partial \mu}{\partial c} \right)_{p,T}} \quad (50)$$

⁴It will be shown later that the diffusion coefficient D_{12} defined here agrees with that appearing in the previous section.

because

$$\nabla\mu = \left(\frac{\partial\mu}{\partial c}\right)_{p,T} \nabla c + \left(\frac{\partial\mu}{\partial T}\right)_{p,c} \nabla T + \left(\frac{\partial\mu}{\partial p}\right)_{c,T} \nabla p$$

where $c = c_1$. Again, since for perfect gas $p = [c/m_1 + (1-c)/m_2] \rho K T$ where $K = \text{Boltzmann's constant}$, the following relations are obtained:

$$\left(\frac{\partial\mu}{\partial c}\right)_{p,T} = \frac{KT}{c(1-c)[cm_2 + (1-c)m_1]} \quad (51)$$

$$\left(\frac{\partial\mu}{\partial p}\right)_{c,T} = \frac{KT}{p} \left(\frac{1}{m_1} - \frac{1}{m_2}\right) \quad (52)$$

$$k_p = (m_2 - m_1)n_1 n_2 (n_1 + n_2) \frac{m_1 m_2}{\rho^3} = \frac{n(m_2 - m_1)\rho_1 \rho_2}{\rho^3} \quad (53)$$

$$k_f = \frac{m_1 m_2 n^2 \rho_1 \rho_2}{\rho^3} \quad (54)$$

Hence

$$\begin{aligned} f_1 = -\rho D_{12} \left[\nabla c + \frac{(m_2 - m_1)n\rho_1 \rho_2}{\rho^3 p} \nabla p - \frac{\rho_1 \rho_2}{p \rho^3} n^2 m_1 m_2 \left(\frac{e_1}{m_1} - \frac{e_2}{m_2} \right) \mathbf{E} \right. \\ \left. + \left(\frac{e_1}{m_1} \langle \mathbf{v}_1 \rangle - \frac{e_2}{m_2} \langle \mathbf{v}_2 \rangle \right) \times \mathbf{B} \right] \quad (55) \end{aligned}$$

Now, if the part concerning ordinary diffusion is rewritten in terms of the gradient of the ratio n_1/n (or n_{10}), then

$$\begin{aligned} f_1 = & - \frac{n^2 m_1 m_2}{\rho} D_{12} \left\{ \nabla n_{10} + \frac{(m_2 - m_1) n_1 n_2}{n \rho p} \nabla p \right. \\ & \left. - \frac{\rho_1 \rho_2}{\rho p} \left[\left(\frac{e_1}{m_1} - \frac{e_2}{m_2} \right) \mathbf{E} + \left(\frac{e_1}{m_1} \langle \mathbf{v}_1 \rangle - \frac{e_2}{m_2} \langle \mathbf{v}_2 \rangle \right) \times \mathbf{B} \right] \right\} \end{aligned}$$

or

$$\begin{aligned} \langle \mathbf{v}_1 \rangle - \langle \mathbf{v}_2 \rangle = & - \frac{n^2}{n_1 n_2} D_{12} \left\{ \nabla n_{10} + \frac{(m_2 - m_1) n_1 n_2}{n \rho p} \nabla p \right. \\ & \left. - \frac{\rho_1 \rho_2}{\rho p} \left[\left(\frac{e_1}{m_1} - \frac{e_2}{m_2} \right) \mathbf{E} + \left(\frac{e_1}{m_1} \langle \mathbf{v}_1 \rangle - \frac{e_2}{m_2} \langle \mathbf{v}_2 \rangle \right) \times \mathbf{B} \right] \right\} \quad (56) \end{aligned}$$

Here it may be seen that the diffusion coefficient D_{12} is the same as that used in Ref. 1 (p. 144). Again, since

$$\begin{aligned} & \left(\frac{e_1}{m_1} - \frac{e_2}{m_2} \right) \mathbf{E} + \left(\frac{e_1}{m_1} \langle \mathbf{v}_1 \rangle - \frac{e_2}{m_2} \langle \mathbf{v}_2 \rangle \right) \times \mathbf{B} \\ & = \left(\frac{e_1}{m_1} - \frac{e_2}{m_2} \right) (\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \left(\frac{e_1}{m_1 \rho_1} + \frac{e_2}{m_2 \rho_2} \right) \mathbf{f}_1 \times \mathbf{B} \end{aligned}$$

Equation (55) may thus be written as

$$\begin{aligned} f_1 = & \frac{n D_{12} (e_1 m_2 \rho_2 + e_2 m_1 \rho_1)}{\rho^2 K T} \mathbf{f} \times \mathbf{B} \\ = & \frac{n^2 m_1 m_2}{\rho} D_{12} \left\{ \nabla n_{10} + \frac{(m_2 - m_1) n_1 n_2}{n \rho p} \nabla p - \frac{\rho_1 \rho_2}{p \rho} \left(\frac{e_1}{m_1} - \frac{e_2}{m_2} \right) (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \right\} \quad (57) \end{aligned}$$

Denoting

$$\frac{nm_1m_2D_{12}}{\rho KT} = \tau$$

$$- \frac{B(e_1m_2\rho_2 + e_2m_1\rho_1)}{\rho m_1m_2} = \omega$$

Eq. (57) becomes

$$\begin{aligned} \mathbf{f}_1 + \omega \tau B^{-1} \mathbf{f}_1 \times \mathbf{B} = & - \frac{n^2 m_1 m_2}{\rho} D_{12} \left[\nabla n_{10} + \frac{(m_2 - m_1) n_1 n_2}{np \rho} \nabla p \right. \\ & \left. - \frac{\rho_1 \rho_2}{\rho p} \left(\frac{e_1}{m_1} - \frac{e_2}{m_2} \right) (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \right] \end{aligned} \quad (58)$$

But, \mathbf{f}_1 is related to \mathbf{J} through the equation

$$\mathbf{J} = \left(\frac{e_1}{m_1} - \frac{e_2}{m_2} \right) \mathbf{f}_1$$

Thus, multiplying all terms in Eq. (58) by $(e_1/m_1 - e_2/m_2)$ and defining

$$\sigma = \frac{n_1 n_2 n}{\rho^2 KT} (e_1 m_2 - e_2 m_1)^2 D_{12}$$

Eq. (58) may be written in the following form:

$$\begin{aligned} \mathbf{J} + \omega \tau B^{-1} \mathbf{J} \times \mathbf{B} = & \sigma \left[(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \frac{m_1 - m_2}{n} \left(\frac{1}{e_1 m_2 - e_2 m_1} \right) \nabla p \right. \\ & \left. - \frac{\rho p}{\rho_1 \rho_2} \frac{m_1 m_2}{(e_1 m_2 - e_2 m_1)} \nabla n_{10} \right] \end{aligned} \quad (59)$$

Using the definition of \mathbf{E}' given in the previous section,

$$\mathbf{J} + \omega \tau \mathbf{B}^{-1} \mathbf{J} \times \mathbf{B} = \sigma \mathbf{E}' \quad (60)$$

Evidently, the solution of \mathbf{J} may be verified to be

$$\mathbf{J} = \sigma \left[\frac{\mathbf{E}' + \omega \tau \mathbf{B}^{-1} \mathbf{B} \times \mathbf{E}' + \omega^2 \tau^2 \mathbf{B}^{-2} (\mathbf{B} \cdot \mathbf{E}')}{1 + \omega^2 \tau^2} \right] \quad (61)$$

which agrees with what has been obtained in Section II.

IV. CONCLUSIONS

It should be pointed out again that the result obtained in the previous discussions manifests only an approximate form of the actual current density, because the analysis has been idealized by many imposed assumptions which simplify considerably both the mathematical task and the physical complexity of the problem. For instance, the investigation is based on classical kinetic theory, in which the collision mechanism is considered to be binary and the coulomb interaction is ignored. These simplifications may cause certain errors; the qualitative picture of the result, however, is believed still significant.

The generalized Ohm's law obtained in previous discussions must be understood to be independent of thermoelectrical effect because the temperature gradient is postulated to be negligible in the previous analyses. Therefore, the so-called Ettingshausen and Nernst effects are not included. Further extension of the discussion along this line should not be very difficult.

The generalized Ohm's law just mentioned reveals one interesting feature which may be stated in the following. If three unit vectors i_1 , i_2 and i_3 are designated in three orthogonal directions E , $(B \times E)$ and $E \times (B \times E)$ respectively and γ is denoted as the angle between B and E , then the current density J may be expressed as

$$J = J_1 i_1 + J_2 i_2 + J_3 i_3$$

where

$$J_1 = \sigma \left(\frac{1 + \omega^2 \tau^2 \cos^2 \gamma}{1 + \omega^2 \tau^2} \right) E'$$

$$J_2 = \sigma \left(\frac{\omega \tau \sin \gamma}{1 + \omega^2 \tau^2} \right) E'$$

$$J_3 = \sigma \left(\frac{\omega^2 \tau^2 \sin \gamma \cos \gamma}{1 + \omega^2 \tau^2} \right) E'$$

J_2 is the generalized Hall current. The Hall conductivity is seen to be a function of the angle γ besides ω and τ (γ is the angle between the vectors B and E). It is interesting to note

that J_3 vanishes in the case \mathbf{B} is either parallel or perpendicular to \mathbf{E} , which are the situations customarily considered in the existing literature.

Finally, it should be remarked that it is not surprising to obtain agreement between the two methods demonstrated in Sections II and III since the fundamental assumptions involved in these methods are equivalent. For example, in Section II the classical Boltzmann equation in kinetic theory is used without considering the coulomb interactions; in Section III, the same assumption is implicitly employed when the derivatives $(\partial\mu/\partial p)_{c,T}$, $(\partial\mu/\partial c)_{p,T}$, etc. are evaluated based on the perfect gas condition. Again, in Section II, discussion of the solution is, up to the second approximation, based on the Chapman-Enskog technique in which the distribution functions of the electrons and ions are assumed to be slightly disturbed from the Maxwellian distributions. This is evidently consistent with the approximations involved in the establishment of the linear phenomenological relations in the theory of irreversible thermodynamics.

REFERENCES

1. Chapman, S., and T. G. Cowling. The Mathematical Theory of Non-Uniform Gases. Cambridge University Press, 1953.
2. Cowling, T. G. Magnetohydrodynamics. New York, Interscience Publishers, Inc., 1957.
3. Spitzer, L. Physics of Fully Ionized Gas. New York, Interscience Publishers, Inc., 1956.
4. Finkelnburg, W. and H. Maecker, "Elektrische Bögen und Thermisches Plasma," in Handbuck der Physik. Berlin, Springer-Verlag, 1956.
5. Jeans, J. H. The Dynamical Theory of Gases, 4th ed. Cambridge University Press, 1925. Chapter X.
6. de Groot, S. R. Thermodynamics of Irreversible Processes, Amsterdam, North Holland Publishing Co., 1951.